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Hall Ticket Number:

Code No.: 11012 O

VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD

B.E. (CBCS) I-Semester Backlog Examinations, December-2017

Engineering Mathematics-I

(Common to all Branches)

Time: 3 hours Max. Marks: 70

Note: Answer ALL questions in Part-A and any FIVE from Part-B

Part-A (10 × 2 = 20 Marks)

1. Find Maclurins expansion for $f(x) = x^3 - 3x^2 + 2x$.
2. Find the radius of curvature at the origin for the curve $2x^3 - 3x^2y + 4y^3 + y^2 - 3x = 0$.
3. If $u = e^{xyz}$ show that $\frac{\partial^3 u}{\partial x \partial y \partial z} = (1 + 3xyz + x^2y^2z^2)\, e^{xyz}$
4. If $x = uv$ and $y = \frac{u+v}{u-v}$ *find* $\frac{\partial(u,v)}{\partial(x,y)}$.
5. Evaluate $\int_0^1 \int_0^x e^{\frac{y}{x}}\, dy\, dx$
6. Evaluate $\iint r^3 dr\, d\theta$ over the area bounded between the circles $r = 2\cos\theta$ and $r = 4\cos\theta$.
7. Find a unit vector normal to the surface $x^3 + y^3 + 3xyz = 3$ at the point (1, 2,-1).
8. Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2,-1, 2).
9. Examine the convergence of the series $1 + \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} + \frac{1}{\sqrt{4}} + \dots\dots\dots + \frac{1}{\sqrt{n}}$.
10. Test the series $1 - \frac{1}{2\sqrt{2}} + \frac{1}{3\sqrt{3}} - \frac{1}{4\sqrt{4}} + \dots\dots\dots$ for convergence and absolute convergence.

Part-B (5 × 10 = 50 Marks)

11. a) Show that $\frac{x}{e^x - 1} = 1 - \frac{x}{2} + \frac{1}{6}\frac{x^2}{2} \dots\dots$ [4]

    b) Find the evolute of the curve $x^{2/3} + y^{2/3} = a^{2/3}$ [6]

12. a) Show that the rectangular solid of maximum volume that can be inscribed in a given sphere is a cube. [5]

    b) Expand $e^x \log(1+y)$ in powers of x and y upto the terms of third degree. [5]

13. a) Evaluate $\iint (x + y)\, dy\, dx$ over the first quadrant of the circle $x^2 + y^2 = 1$. [5]

    b) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$ [5]

14. a) If the vector $\mathbf{F} = (ax^2y + yz)\,\mathbf{i} + (xy^2 - xz^2)\,\mathbf{j} + (2xyz - 2x^2y^2)\,\mathbf{k}$ is solenoidal, find the value of 'a' and also the curl of this solenoidal vector. [5]

Contd... 2

b) Evaluate $\iint_S \vec{F}.\hat{n}\,ds$ where F = z i + x j - $3y^2z$ k and S is the surface of the cylinder $x^2 + y^2 = 16$ included in the first octant between z = 0 and z = 5. [5]

15. a) Test the convergence and absolute convergence of the series [5]
$\frac{1}{2(\log 2)^p} + \frac{1}{3(\log 3)^p} + \frac{1}{4(\log 4)^p} + \ldots\ldots$ (p > 0)

b) Test the convergence of the series $\sum_{n=2} \frac{1}{n(\log n)^p}, p > 0$ [5]

16. a) Find the envelope of the two parameter family of parabola $\sqrt{\frac{x}{a}} + \sqrt{\frac{y}{b}} = 1$ where the two parameters a and b are connected by the relation a + b = c, and c is a given constant. [5]

b) Divide 120 into three parts so that the sum of their products taken two at a time shall be maximum. [5]

17. Answer any *two* of the following:

a) Evaluate $\iiint x^2yz\,dx\,dy\,dz$ over the region bounded by the planes x = 0, y = 0, z = 0 and x + y + z = 1. [5]

b) Apply Green's theorem to evaluate $\int_C \left[(2x^2 - y^2)\,dx + (x^2 + y^2)\,dy\right]$ where C is the boundary of the area enclosed by the x-axis and the upper-half of the circle $x^2 + y^2 = a^2$. [5]

c) Test the convergence of the series $\frac{3}{7}x + \frac{3\times6}{7\times10}x^2 + \frac{3\times6\times9}{7\times10\times13}x^3 + \ldots\ldots\ldots(x > 0)$. [5]

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VASAVI COLLEGE OF ENGINEERING *(Autonomous)*, HYDERABAD
B.E. (CBCS) I-Semester Main Examinations, December-2017
Engineering Mathematics-I
(Common to all branches)

Time: 3 hours Max. Marks: 60

*Note: Answer **ALL** questions in **Part-A** and any **FIVE** from **Part-B***

Part-A (10 × 2 = 20 Marks)

1. Verify Taylor's theorem for $f(x)=x^3-3x^2+2x$ in $\left[0,\frac{1}{2}\right]$ with Lagrange's remainder up to two terms.
2. Find the radius of curvature at the origin for the curve $2x^3-3x^2y+4y^3+y^2-3x=0$.
3. If $u=e^{xyz}$ show that $\frac{\partial^3 u}{\partial x\partial y\partial z}=(1+3xyz+x^2y^2z^2)\,e^{xyz}$
4. If $u=x^2-y^2,\ v=2xy$ where $x=r\cos\theta,\ y=r\sin\theta$ find $\frac{\partial(u,v)}{\partial(r,\theta)}$.
5. Evaluate $\int_0^1\int_0^{\sqrt{1+x^2}}\frac{dydx}{1+x^2+y^2}$
6. Evaluate $\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta$
7. Find a unit vector normal to the surface $x^3+y^3+3xyz=3$ at the point (1,2,-1).
8. Find the angle between the surfaces $x^2+y^2+z^2=9$ and $z=x^2+y^2-3$ at the point (2,-1,2).
9. Examine the convergence of the series $\frac{3}{5}+\frac{4}{5^2}+\frac{3}{5^3}+\frac{4}{5^4}+\ldots\ldots\ldots$
10. If $\sum u_n$ is a series of positive terms then prove that $\lim_{n\to\infty}u_n=0$.

Part-B (5 × 8 = 40 Marks)
(All bits carry equal marks)

11. a) Expand $e^{\sin x}$ by Maclaurin's series up to the term containing x^4.

    b) Find the coordinates of the centre of curvature at any point (x,y) on the parabola $y^2=4ax$. Also find the equation of the evolute of the parabola.

12. a) Find the extreme values of the function $x^3+y^3-3x-12y+20$.

    b) Find the minimum and maximum distance from the point (1, 2, 2) to the sphere $x^2+y^2+z^2=36$.

Contd...2

13. a) Evaluate $\iint \sqrt{\frac{1-x^2-y^2}{1+x^2+y^2}}\, dx\, dy$ over the first quadrant of the circle $x^2+y^2=1$.

b) Evaluate $\int_0^a \int_0^x \int_0^{x+y} e^{x+y+z}\, dz\, dy\, dx$

14. a) If the vector $F= (ax^2y+yz)\, i + (xy^2-xz^2)\, j + (2xyz-2x^2y^2)\, k$ is solenoidal, find the value of 'a' and also the curl of this solenoidal vector.

b) Evaluate $\iint_s \vec{F}\cdot\hat{n}\, ds$ where $F=z\,i + x\,j - 3y^2z\,k$ and S is the surface of the cylinder $x^2+y^2=16$ included in the first octant between z=0

15. a) Discuss the convergence of $\frac{1}{2\sqrt{1}}+\frac{x^2}{3\sqrt{2}}+\frac{x^4}{4\sqrt{3}}+\frac{x^6}{5\sqrt{4}}+\text{.........},(x>0)$

b) Test the series $\frac{x}{\sqrt{3}}-\frac{x^2}{\sqrt{5}}+\frac{x^3}{\sqrt{7}}-\text{.........}$ for absolute and conditional convergence

16. a) Show that the radius of curvature at any point of the curve $x^{2/3}+y^{2/3}=a^{2/3}$ is equal to three times the length of the perpendicular from the origin to the tangent at that point.

b) If $z=f(u,v)$, $u=\log(x^2+y^2)$, $v=y/x$, show that $x\frac{\partial z}{\partial y}-y\frac{\partial z}{\partial x}=(1+v^2)\frac{\partial z}{\partial v}$.

17. Answer any *two* of the following:

a) Evaluate $\iiint x^2yz\, dx\, dy\, dz$ over the region bounded by the planes x=0, y=0, z=0 and x+y+z=1.

b) Apply Green's theorem to evaluate $\int_C \left[(2x^2-y^2)\,dx+\left(x^2+y^2\right)dy\right]$ where C is the boundary of the area enclosed by the x-axis and th e upper-half of the circle $x^2+y^2=a^2$.

c) Test the convergence of the series
$\frac{3}{7}x+\frac{3\times 6}{7\times 10}x^2+\frac{3\times 6\times 9}{7\times 10\times 13}x^3+\text{..........}(x>0)$.

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